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Effective Field Theories for Heavy Quarkonium

N. Brambilla[*]

 Dipartimento di Fisica dell' Universitá di Milano and INFN, via Celoria 16, 20133 Milano, Italy

Abstract. We briefly review how nonrelativistic effective field theories give us a definition of the QCD potentials and a coherent field theory derived quantum mechanical scheme to calculate the properties of bound states made by two or more heavy quarks. In this framework heavy quarkonium properties depend only on the QCD parameters (quark masses and $\alpha_{\rm s}$) and nonpotential corrections are systematically accounted for. The relation between the form of the nonperturbative potentials and the low energy QCD dynamics is also discussed.

1 The Physical System

Hadron properties should be obtained from the QCD Lagrangian as a function of the coupling constant $\alpha_{\rm s}$ and of the quark masses m. In practice, things are made complicate by QCD being a strongly coupled theory in the low energy region. At the scale $\Lambda_{\rm QCD}$, nonperturbative effects become dominant and $\alpha_{\rm s}$ becomes large. The nonperturbative QCD dynamics originates the confinement of quarks inside hadrons. Typical approaches include, on one hand the use of phenomenological potential models and constituent quark model descriptions, on the other hand first principles lattice simulations (still far from the physical parameter window in many cases). However, the physics of systems with a heavy quark Q allows some simplification. The quark mass scale m_Q is large, bigger than $\Lambda_{\rm QCD}$. Then $\alpha_{\rm s}(m_Q)$ is small and perturbative expansions may be performed at this scale. Bound systems made of two or more heavy quarks are even more interesting [1]. They are nonrelativistic systems characterized by another small parameter, the heavy-quark velocity v, and by a hierarchy of energy scales: m_Q (hard), the relative momentum $p \sim m_Q v$ (soft), and the binding energy $E \sim m_Q v^2$ (ultrasoft). For energy scales close to $\Lambda_{\rm QCD}$, perturbation theory breaks down and one has to rely on nonperturbative methods. Regardless of this, the nonrelativistic hierarchy $m_Q \gg m_Q v \gg m_Q v^2$ persists also below the $\Lambda_{\rm QCD}$

[*]*E-mail address:* nora.brambilla@ptt.mi.infn.it

threshold. While the hard scale is always larger than Λ_{QCD}, different situations may arise for the other two scales depending on the considered quarkonium system. The soft scale, proportional to the inverse typical radius r, may be a perturbative ($\gg \Lambda_{\mathrm{QCD}}$) or a nonperturbative scale ($\sim \Lambda_{\mathrm{QCD}}$) depending on the physical system. The ultrasoft scale may still be perturbative only in the case of $t\bar{t}$ threshold states.

2 Scales and EFTs: down to pNRQCD

Taking advantage of the existence of a hierarchy of scales, one can introduce nonrelativistic effective field theories (NR EFTs) [2] to describe heavy quarkonia. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to high energy scales not relevant for quarkonium. Such integration is made in a matching procedure enforcing the equivalence between QCD and the EFT at a given order of the expansion in v ($v^2 \sim 0.1$ for $b\bar{b}$, $v^2 \sim 0.3$ for $c\bar{c}$, $v \sim 0.1$ for $t\bar{t}$). The EFT realizes a factorization at the Lagrangian level between the high energy contributions, encoded into the matching coefficients, and the low energy contributions, carried by the dynamical degrees of freedom. Poincaré symmetry remains intact in a nonlinear realization at the level of the NR EFT and imposes exact relations among the matching coefficients [3, 4].

By integrating out the hard modes one obtains Nonrelativistic QCD [5, 6, 4]. NRQCD is making explicit at the Lagrangian level the expansions in $m_Q v/m_Q$ and $m_Q v^2/m_Q$. It is is similar to HQET, but with a different power counting and accounts also for contact interactions between quarks and antiquark pairs (e.g. in decay processes), hence having a wider set of operators. In NRQCD soft and ultrasoft scales are dynamical and their mixing may complicate calculations, power counting and do not allow to obtain a Schrödinger formulation in terms of potentials. One can go down one step further and integrate out the soft scale in a matching procedure to the lowest energy EFT that can be introduced for quarkonia, where only ultrasoft degrees of freedom remain dynamical. Such EFT is called potential NonRelativistic QCD (pNRQCD) [7, 8]. In this case the matching coefficients encode the information on the soft scale and are the potentials. pNRQCD is making explicit at the Lagrangian level the expansion in $m_Q v^2/m v_Q$. This EFT is close to a Schrödinger-like description of the bound state. The bulk of the interaction is carried by potential-like terms, but non-potential interactions, associated with the propagation of low-energy degrees of freedom ($Q\bar{Q}$ colour singlets, $Q\bar{Q}$ colour octets and low energy gluons), are generally present. They start to contribute at NLO in the multipole expansion of the gluon fields and are typically related to nonperturbative effects [8].

In this EFT frame, it is important to establish when Λ_{QCD} sets in, i.e. when we have to resort to non-perturbative methods. For low-lying resonances, it is reasonable to assume $m_Q v^2 \gtrsim \Lambda_{\mathrm{QCD}}$. The system is weakly coupled and we may rely on perturbation theory, for instance, to calculate the potential. In this case, we deal with weak coupling pNRQCD. The theoretical challenge here is performing higher-order perturbative calculations and the goal is precision physics (see e.g. [9, 2]). For higher resonances $m_Q v \sim \Lambda_{\mathrm{QCD}}$. In this case, we deal with

strongly coupled pNRQCD. We need nonperturbative methods to calculate the potentials and one of the goal is the investigation of the QCD low energy dynamics.

3 The QCD potentials

pNRQCD [7, 8] realizes modern renormalization theory in the context of simple nonrelativistic Quantum Mechanics [10]. In this framework the Schrödinger equation is exactly the equation to be solved to get the binding. The $Q\bar{Q}$ potentials to be used in such equation are the Wilson matching coefficients of pNRQCD obtained by integrating out from QCD all degrees of freedom but the ultrasoft ones.
Perturbative Potentials
If the quarkonium system is small, the soft scale is perturbative and the potentials can be *entirely* calculated in perturbation theory [2]. They undergo renormalization, develop a scale dependence and satisfies renormalization group equations, which eventually allow to resum potentially large logarithms. Since the degrees of freedom that enter the Schrödinger description are in this case both $Q\bar{Q}$ color singlet and $Q\bar{Q}$ color octets, both singlet and octet potentials exist. At the moment, the static singlet $Q\bar{Q}$ potential is known at three loops apart from the constant term. The first log related to ultrasoft effects arises at three loops. Such logarithm contribution at N^{3}LO and the single logarithm contribution at N^{4}LO may be extracted respectively from a one-loop and two-loop calculation in the EFT and have been calculated in [11, 12]. The singlet static energy, given by the sum of a constant, the static potential and the ultrasoft corrections, is free from ambiguities of the perturbative series (renormalon). By comparing it (at the NNLL) with lattice calculations of the static potential one sees that the QCD perturbative series converges very nicely to and agrees with the lattice result in the short range and that no nonperturbative linear ("stringy") contribution to the static potential exist [13, 2].

The $1/m_Q$ singlet potential is known at two loops, the $1/m_Q^2$ singlet spin-dependent and spin-indepent potentials are known at one loop (see the discussion and the original references quoted in [2, 15, 14]).
Nonperturbative potentials

If the quarkonium system is large, the soft scale is nonperturbative and the potentials cannot be entirely calculated in perturbation theory [2]. They come out factorized in the product of NRQCD matching coefficients and low energy nonperturbative parts given in terms of Wilson loops expectation values and field strengths insertions in the Wilson loop. The full expression for the QCD potentials up to order $1/m_Q^2$ has been obtained in [16], for the QQQ and QQq case see [17]. Such expressions correct and generalize previous findings in the Wilson loop approach [18] that were typically missing the high energy parts of the potentials, encoded into the NRQCD matching coefficients and containing the dependence on the logarithms of m_Q, and some of the low energy contributions. Poincaré invariance establishes exact relations between the potentials [3] of the type of the Gromes relation between spin dependent and static potentials [19].

In this regime, *from pNRQCD we recover the quark potential singlet model.*

However, here the potentials are calculated from QCD by nonperturbative matching. Their evaluation requires calculations on the lattice [20] or in QCD vacuum models [21, 22]. Recently lattice calculations have reached a high degree of precision [23] and well defined predictions on the behaviour of the $1/m_Q$ and $1/m_Q^2$ potentials in the confining region became possible. Such behaviour seems to deviate from a flux tube picture [22].

Since now precise lattice data on the long distance behaviour of the potentials up to order $1/m_Q^2$ are available, one should relate them to QCD vacuum model predictions on expectation values of Wilson loops and field strengths Wilson loop insertions. These are gauge invariant objects containing information on the field distribution between the quarks that go well beyond the area law content. They give us an appropriate way to "measure" and characterize the confinement mechanism. For this reason it would be interesting to get predictions on the behaviour of this objects also from string theory.

4 How to obtain the Spectra

When the soft scale is perturbative the energy levels are given by the expectation value of the perturbative potentials, calculated at the needed order of the expansion in α_s, plus nonpotential nonperturbative contributions [24, 14, 2]. The latter start to contribute to the energy levels at order $m_Q \alpha_s^5$. They are retardation effects and are systematically accounted for in the EFT. They enter energy levels (and decay widths) in the form of local or nonlocal electric and magnetic condensates. We still lack a precise and systematic knowledge of such nonperturbative purely glue dependent objects. It would be important to have for them lattice determinations or data extractions. Inside pNRQCD it is possible to relate the leading electric and magnetic nonlocal correlators to the gluelump masses and to some existing lattice (quenched) determinations [2]. However, since the nonperturbative contributions are suppressed in the power counting, it is possible to obtain precise determinations of the masses of the lowest quarkonium resonances with purely perturbative calculations, in the cases in which the perturbative series is convergent (i.e. after that the appropriate subtractions of renormalons have been performed) and large logarithms are resummed. In this framework power corrections are unambiguously defined. For a review of the predictions on the lowest quarkonium resonances obtained in this framework see [2, 9].

When the soft scale is nonperturbative the energy levels are given by the expectation values of the nonperturbative potentials described in the previous section. *Nonpotential (or retardation) corrections do not exist in this case.* A full phenomenological application, taking into account both the NRQCD matching coefficients and the recent lattice evaluation of the low energy part of the potentials, has not yet been performed and would be needed.

In both cases, the EFT supply us with a proper and well defined quantum mechanical framework to perform systematic calculations of the quarkonium spectrum. In particular:

- There is a well defined power counting that states the terms that should be treated as (quantum mechanical) perturbation.

- In higher-order calculations, quantum mechanical perturbation theory requires regularization and renormalization. pNRQCD gives us a well defined and field theory derived quantum mechanical framework to calculate perturbative corrections. In particular, the soft UV divergences in the potential cancel against NRQCD hard matching coefficients [25, 2] and potential UV divergences in quantum mechanical perturbation theory cancel against NRQCD hard matching coefficients [26, 2], leaving well behaved and scale independent predictions for physical quantities. Then, in this scheme no divergences arise from e.g. the iteration of the spin-spin potential in quantum mechanical perturbation theory, as it happens typically in phenomenological potential model approaches..

- Spectra are function only of the Standard Model parameters. Conversely one can use the spectra in order to extract α_s and m_Q [9].

5 Decays and Transitions

While the real parts of the pNRQCD matching coefficients give us the potentials, the imaginary parts give us the inclusive decays widths [27]. Also transitions may be worked out in pNRQCD. In [28] the M1 transition rates for the lowest quarkonia resonances has been calculated and a value for $\Gamma(J/\psi \to \gamma\eta_c)$ compatible with the experimental data has been obtained.

6 States close to threshold

The results on the nonperturbative potentials that we have discussed are valid away from threshold and in the case in which hybrids develop a mass gap of order Λ_{QCD} with respect to singlet states [2], as the lattice indicates. For states near or above threshold a general systematic EFT approach has still to be developed, while lattice results on excited resonances are just appearing. Most of the existing analyses, especially for the many new states discovered recently at the B factories, have to rely on a formalism based on potential models and coupled channels, see e.g. [29] and refs. therein.

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